Reprise: The Meaning of Conservation Laws

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A few years ago a paper appeared in *TPT* on the topic of conservation laws.[1] Perhaps it is only the increased sensitivity of age, but it seems there has been a serious regression in the intervening decades in choice of terminology regarding conservation.

We may agree at the outset that we are restricting the term to the scientific meaning of the conservation laws, ignoring political, ecological, or other distractions. Nevertheless, we find in the recent scientific literature examples of quantities that have been described as being conserved that include velocity, energy, mechanical energy, kinetic energy, heat, work, current, volume, and entropy.[2] We might begin to suspect that authors are using the term in different ways.

By generally accepted standards,[3]

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[1] Robert P. Bauman, "The Meaning of Conservation Laws", *Phys. Teach.* **9**, 186-189 (April, 1971).

[2] The intent here is not to criticize those who have misunderstood our textbooks, *e.g.*, by confusing constants of the motion with quantities subject to conservation laws.. Therefore specific references are limited to publications that have made a positive contribution to the topic.

[3] The most common statement is, *e.g.*, of the form "the energy of the universe is constant." That was satisfactory in the 19th century, but has been questioned as we learn how little we know about the edges of the universe. The alternative form here is more "conservative" (*i.e.*, it makes fewer assumptions).

Newton was careful to avoid energy (which was not carefully defined until long after his death). Leibniz, Black, Lavoisier, Rumford, Young, Carnot, and Coriolis contributed to the early understanding of energy and entropy, incorporated eventually into thermodynamics, which has been a foundation stone of physics for a little more than a century and a half.

A key to the process of applying thermodynamics is recognition that one must identify what body, or "system", possesses or receives the quantities transferred between objects. Hence the strong emphasis on distinguishing "system" from "surroundings". The specific wording chosen here emphasizes the need to avoid problems of cosmology when the focus is on local (or "laboratory scale") measures.

It is also widely recognized that any conservation law is easily subject to overthrow because a single confirmable exception is sufficient to refute such a law. It is indeed remarkable that conservation laws have been able to withstand *every* experimental attempt to demonstrate their violation. We can only test such a law by testing it over and over, in a wide variety of circumstances.

a quantity is said to be "conserved" if, *for every possible process*, the amount is constant *for the universe* or, somewhat less presumptuously, constant for the system and the surroundings, where the "surroundings" include everything in the universe that might be affected by the process under consideration.

Under this definition there are seven quantities that are conserved in mechanics: energy, momentum (three components), and angular momentum (three components). To these we should really add volume, because any change in volume of the system is necessarily equal and opposite to the change in volume of the surroundings. (This is surely the easiest to see and recognize, and should "lead the parade" when we explain conservation to a new audience.)[4] We probably also should add mass, although conservation of mass is not independent. Conservation of mass is already included in the law of conservation of energy, because mass is nothing but a measure of total energy[5]; $m = E/c^2$.

Despite wide-spread lip service to these conservation laws, we still find frequent references to "provisional" conservation. We read that energy is conserved if ⋯, momentum is conserved when ⋯, or angular momentum is conserved subject to ⋯. Similarly, we find frequent reference to the concept that the energy *of the system* is constant because of conservation of energy, when there has been no consideration of how or whether energy is transferred to or from the surroundings.

It appears there is a serious need for a better term to describe "constant energy for the system" to cover the estimated 90% of the problems in which conservation of energy is not the issue. For example, textbooks also quote a "law of conservation of mechanical energy", although we know *only* total energy is conserved. Mechanical energy may be constant for a system, under appropriate circumstances, but that does not cause mechanical energy to be conserved. If conservation of energy were the intended meaning, it would not be necessary to specify limitations, such as absence of friction or other mis-named "nonconservative" forces. In another context, it has been argued that mass is not *conserved* because the mass of an object is not *constant*, a direct confusion of "conservation" with a "constant of the motion".

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[4] Volume is well defined within "laboratory scale" measurements. Through confusion of terminology, constancy of volume of a liquid as it is poured between vessels has been called "conservation of volume". If the liquid is warmed, or if a block of copper is warmed, or a balloon is expanded, the volume *of the system* changes, but any increase in the volume of the system comes at the expense of volume of the surroundings. (For example, the volume of the room is constant; only the division between [volume of system] and [volume of room - system = volume of surroundings] changes.) More detailed arguments have been given in Robert P. Bauman, *A First Course in Physical Science*, Wiley, New York, 1987; chapter 7, 20, and 21, and in Robert P. Bauman, *Modern Thermodynamics with Statistical Mechanics*, Macmillan, New York, 1992; chapter 21.

[5] Einstein's basic equation, $E = mc^2$, tells us (even after 100 y) that however we measure mass, it is a measure of the *total energy* (something not recognized as existing in the classical mechanics of the 19th century). A direct consequence is that E must always be a positive quantity so long as m is positive.

A description that appears to work well, without causing unnecessary confusion or obscure language, is the following:

A quantity is *preserved* when it remains constant for the system.

In elastic collisions, frictionless systems, and many other common problems, energy is preserved. When there are no external forces acting, momentum is preserved. If there are no torques acting on the system, angular momentum of the system is preserved. When these conditions are *not* met, energy, momentum, and angular momentum are (necessarily still) *conserved*, but these quantities are no longer constants of the motion.

A short cut often attempted is to express conservation laws in a form that says the quantity is constant (*i.e.*, preserved) for an isolated system. Unfortunately, isolated systems are not defined. It is not clear whether one is to keep volume constant and/or pressure or force of some other quantity zero or constant. And, of course, it is not possible to carry out any experiment on an isolated system. Certainly the contents of a rigid container in the laboratory, under atmospheric pressure and balanced gravitational and tension or compression forces, exchanging thermal energy, even slowly, with the surroundings, should not be considered “isolated”. (Unless the intention is to say that a body that does not exchange energy or momentum or volume or charge or other quantities with its surroundings does not exchange energy or momentum or volume or charge or other quantities with its surroundings.)

Constants of the motion are important quantities, even in problems in which there is no motion and the label does not properly fit. To describe these situations as subject to *preservation rules* is economical and precise.